BULETINUL Universității Petrol – Gaze din Ploiești	Vol. LX No. 2/2008	115-121	Seria Matematică - Informatică - Fizică

The Problem of Relation between Energy and Mass

Ion Simaciu, Zoltan Borsos, Mihai Hotinceanu

Universitatea Petrol-Gaze din Ploieşti, Bd. Bucureşti 39, Ploieşti, 100680
e-mail: isimaciu@yahoo.com

Abstract

In this paper, we analyze the concepts about two fundamental physical measures: the energy and the mass. In first part, we presented these concepts from the classical physics. In part two, are presented the concepts from the modern physics: the relativistic physics and the quantum physics. In part three, are presented new arguments to support the fact that the measures are qualitative distinct and the Einstein relation is just a quantitative relation between mass in the proper referential system and energy. The key of the problem solution is a relativistic definition of the speed in connections with the relativistic definitions the path length and the duration of the path. The relativistic definitions are compatible only in the case of proper speed $\vec{\upsilon} = d\vec{r}/d\tau$ *. In this case, the relativistic momentum is the product between the mass in proper reference and proper velocity* $\vec{p} = m\vec{\upsilon}_\tau = m\vec{\upsilon}/\sqrt{1-\upsilon^2/c^2}$.

Keywords: *energy, inertial mass, relativistic mass, proper mass*

Introduction

The energy is the physical measure that measures the property the physical system to modify the physical state (the relative rest, external or internal motion, the deformation etc.) of another system with that he interacts. The energy of the system is a measure/state of function (for instance the kinetic energy E_k). The energy changes of the system is measured depending on process (the mechanic work L, the heat Q).

The material systems are modeled by the ensembles of subsystems interacted each with others. According to the nature of the subsystems and the type of interaction, the energy can be: *thermal mechanic, electromagnetic, nuclear, gravitational, chemical, biological* etc. In physics is postulated that the *energies: mechanic, thermal, chemical, biological,* are types of energies that can be explicated with help of fundamental interactions: *electromagnetic, weak* (after unification of the theories, electro - weak), *gravitational and nuclear and the proper energies of these interactions*. In the popularization literature of the relativistic theory and the theory of elementary particles we find, often, assertions of the kind: „Einstein relation $E = mc^2$ evidence the fact that the energy is can changed into mass", „at annihilation of particle and his antiparticle, these are transformed integral in energy" or other equivalents. To identify the errors from these assertions must be define the notion of mass.

The mass is the physical measures that measure the property of the physical system to oppose the relative modification his rest state or rectilinear and uniform motion. This mass is known under name of *inertial mass*. The mass measures the property of the physical system to generate gravitational field (to attract gravitational another physical systems). This type of mass is known under the name of *gravitational mass* [1]. The definitions do the clear distinction between system and the measures that measure the property of the systems (the measures/the parameters of state). In this distinction, we can conclude easy the fact that his Einstein relation $E = mc^2$ is a quantitative relation that evidence the fact that, if a material system has an amount of energy E, than the system has and an inertial and gravitational mass m proportional with that energy (in the accepted conception of the relativistic mass notion). The equality is referred to the quantitative aspect of the relation and not to qualitative one. Also, *we can't does the assertion that the energy has mass*. Correct is to say that any system that has energy has relativistic mass. In the case of the assertion that the system particle -antiparticle is annihilated in pure energy, can assert, in the light of the definitions enounced above, that is done the error confounded the system with his property. There are omitting of the fact that after annihilation is formed photons (the quanta of the electromagnetic field) that take over the energy of the particles and the antiparticles. Therefore the matter do not is changed to his energy or is composed from energy. *The material systems having a certain energy and mass are transformed through interaction in another material systems having same amount of energy and relativistic mass*. In the respect conservation of the mass, the views are different. Physicists that accepts the notions of relativistic mass and rest mass considers that the photons – null rest mass – in this process the matter is transformed in energy. In their conception, the photons have relativistic mass and this (the energy of the photons divided to square the speed of the light in vacuum) is equal with the relativistic mass of the system of particle - antiparticle (the conservation of the relativistic mass is involved by the conservation of the energy, through the proportionality between the relativistic mass and energy). For a large discussion about this see, Lev Okun [2] and Gary Oas [3].The physicists that reject the notion of relativistic mass accept the conservation of the energy and the conservation of mass of the systems composed by two types of particles. As part as the theory of relativity is demonstrated [4] that the photon has no mass but a system of two photons has mass depends on the angle between these momentums (mass is null when the photons are moved in same direction and is maximum when is moved in opposite directions). Because in the reaction of annihilation results at least two moving photons in different directions, mass the system of the photons is equal with the mass of the system of particles (if the particle and the antiparticle are moving, the system mass is bigger than the sum of their mass, considered separated). This position on materiality of fields, i.e. of the electromagnetic field, is sustained also by the conception expressed through generalized Mach Principle [5].

We consider that this difficult problem shall be clarified when problem of the generating „mechanism" of the mechanical inertia will be resolved and by default the gravitational interaction. The same „mechanisms" were suggested in the theories initiated by Higgs [6,7] and by Haisch, Rueda and Puthoff as part of the stochastic physics [8,9].

The Relation Between the Energy and Mass in Classical Physics

The dynamics of material point (particle) introduces through the process law (the fundamentally law: the relation between the measure of the effect and the measure of the cause)

$$\vec{a} = \vec{F}/m, \tag{1}$$

measure of the mechanical inertia, in the translation motion, which is the inertial mass. The inertial mass appears at the denominator of the fundamental law.

The parameter of dynamic state, the momentum $\vec{p}$ of the particle, measures the „quantity" of translation motion. The momentum, by definition, depends on mass and of the speed vector of the material point, as follows:

$$\vec{p} = m\vec{\upsilon}. \tag{2}$$

The kinetic energy of a material point

$$E_c = m\left(\upsilon^2/2\right) = p^2/\left(2m\right) \tag{3}$$

depend on mass, speed and the mometum [10].

The Relation between the Energy and Mass in Modern Physics

In the relativistic physics, are two ways to interpret the relations between energy and mass. The interpretation with eldest spread among physicists is that in which the mechanical inertia and therefore the mass depend on the speed of material point, as per the relation

$$m = m_0\Big/\sqrt{1-\upsilon^2/c^2}\ . \tag{4}$$

In this relation m is the *relativistic mass* and m_0 is *rest mass* or *proper mass* and υ is the module of the speed vector ($d\vec{r}/dt$, the relative speed) in report with the reference system in that the particle moveing.

The energy of the particle is

$$E = mc^2 = m_0c^2\Big/\sqrt{1-\upsilon^2/c^2} = E_0\Big/\sqrt{1-\upsilon^2/c^2} \tag{5}$$

and $E_0 = m_0c^2$ is the rest energy.

The relativistic momentum has same definition (2) in which m is relativistic mass (1)

$$\vec{p} = m\vec{\upsilon} = m_0\vec{\upsilon}\Big/\sqrt{1-\upsilon^2/c^2}\ . \tag{6}$$

The energy of the particle is expressed depending on the momentum, by the relation

$$E = \sqrt{\left(m_0c^2\right)^2 + p^2c^2} = \sqrt{E_0^2 + p^2c^2}\ . \tag{7}$$

In the relativistic dynamics of material point, the fundamental law is written

$$\mathrm{d}\vec{p}/\mathrm{d}t = \vec{F}\ , \tag{8}$$

with $\vec{p}$ given by the relation (6).

For the physicists specialized in the relativity theory and which I use the Minkowski formalism [11], mass the physical system m, is invariant (he don't depends on the system of inertial reference and therefore of speed). The spatial part of the four-vector of the momentum has the expression

$$\vec{p} = m\vec{\upsilon}_\tau\ , \tag{9}$$

in which $\vec{\upsilon}_\tau = \mathrm{d}\vec{r}/\mathrm{d}\tau$ is the *speed in the proper time* and m the *proper mass*. Because $\mathrm{d}\tau = \mathrm{d}t\sqrt{1-\upsilon^2/c^2}$, the speed in the proper time has the expression $\vec{\upsilon}_\tau = \vec{\upsilon}\Big/\sqrt{1-\upsilon^2/c^2}$ depending on the relative speed and therefore the relativist momentum is given by the expression (6) with the notational change $m_0 \to m$.

Einstein relation between energy and mass is applied merely for the rest energy (the energy in the proper reference system)

$$E_0 = mc^2 \tag{10}$$

and the expression of the energy in report with the system of reference respect to the particle is moving (7) with the notational change $m_0 \to m$.

For a system of two free particles with the energy and the respective momentum $E_1, \vec{p}_1$ and $E_2, \vec{p}_2$, in accord to the energy and momentum conservation law, the energy and the momentum of the system are

$$E = E_1 + E_2 \,,\ \vec{p} = \vec{p}_1 + \vec{p}_2 \,. \tag{11}$$

Using the relation (7) in which we replace equations (11), the system mass is

$$m = \sqrt{\left(E^2/c^4\right) - \left(p^2/c^2\right)} = \sqrt{m_1^2 + m_2^2 + 2\left[\left(E_1 E_2/c^4\right) - \left(p_1 p_2 \cos\alpha/c^2\right)\right]} \,. \tag{12}$$

For a system of two photons, that have null mass, the relation (12) he becomes

$$m_{s.ph} = \sqrt{2E_1 E_2 \left(1-\cos\alpha\right)}\Big/c^2 = \sqrt{2 p_1 p_2 \left(1-\cos\alpha\right)}\Big/c \neq 0 \,. \tag{13}$$

Results that a system of photons has a non – zero mass, although the photons are particles with null mass [4].

New Arguments in the Support of Definition of the one Type of Mass

The option of the experts in the relativity theory for a single type of mass is determined by the invariance requirements imposed by the quadri-dimensional formalism.

In follow, we treat a new involved argument in the relativistic definitions of the measures. In the relativity theory distinguish two types of measurements: *proper measurements* and *relative/mixed/criss-cross measurements*. Proper measurements are the measurements in which as much the object O and the instrument I (by default the subject S effectuate the measurement) are in same conditions (same fields, same types motion etc.) that define the reference or referential (R). The results of this type of measurement are *the proper measurement*. The relative measurements are the measurements in which O and I (by default S) are in different conditions, with another terms, they are in different referential. The results of this type of measurements are *the relative measures* [11,12]. We analyze the kinematic parameter speed and dynamical parameter momentum from relativistic view of point. These parameters are derivative measures. As per the definition, the speed is the ratio between the distance and the duration to path this distance. *The path* is a line segment with length as *the difference of the extremities coordinates in same moments*. The duration the phenomenon is defined as the difference between the final moment (the terminative moment of the phenomenon) and the initial moment (the moment begin the phenomenon), measured in same point place. *The motion phenomenon is a special phenomenon, from viewpoint of these requirements of the relativistic of definitions of the parameter in report with two inertial referential*. For the motion phenomenon, the segment path is different from zero just in report with the fixed referential, in report wherewith the mobile (the moving object) is moved. The duration of the phenomenon can it measure as much in the fixed referential (*the relative duration*) or in the moving referential (in the proper system, *the proper time*). For this reason can define two speeds. For a infinitesimally duration a motion and an infinitesimal path, define a *relative speed* $\mathrm{d}\vec{r}$ (coordinate velocity):

$$\vec{\upsilon} = \mathrm{d}\vec{r}/\mathrm{d}t \tag{14}$$

and a speed in the proper time

$$\vec{\upsilon}_\tau = \mathrm{d}\vec{r}/\mathrm{d}\tau = \vec{\upsilon}\Big/\sqrt{1-\upsilon^2/c^2} \,. \tag{15}$$

In the relations (14) and (15), $\mathrm{d}t$ i*s the infinitesimal relative duration* and $\mathrm{d}\tau$ is *the infinitesimal proper duration* (the infinitesimal per time - the invariant time) [13]. We can define and a proper speed $\vec{\upsilon}_{p\tau} = \mathrm{d}\vec{\rho}/\mathrm{d}\tau$ and a relative proper speed $\vec{\upsilon}_p = \mathrm{d}\vec{\rho}/\mathrm{d}t$ there are nulls, because the proper path $\mathrm{d}\rho$ is zero. From the two speeds non-zero, the speed in proper time is that a speedometer measures attached on the system of locomotion [13]. We evidenced just that this speed respects the relativistic condition as *the duration of the phenomenon measured by a clock positioned in same place.* Results that, from four speeds that we can them define using four measures $\mathrm{d}\vec{r}, \mathrm{d}\vec{\rho}, \mathrm{d}t, \mathrm{d}\tau$, *the only one that carries out as much the definition of speed how much the relativistic definition of duration of the phenomenon is the speed in the proper time*.

With these relativistic limitations, t*he mobile momentum can be defined merely with help of the speed in proper time*, according to the relation (6), with the notational change $m_0 \to m$.

From same reason/considerations, the acceleration that carries out all criterions is the *acceleration in proper time*

$$\vec{a}_\tau = \mathrm{d}\vec{\upsilon}_\tau / \mathrm{d}\tau . \tag{16}$$

Between this acceleration and the vectorial component of the four-vector force in proper time exists the relation

$$\vec{a}_\tau = \vec{F}_\tau / m \text{ or } \mathrm{d}\vec{p}_\tau / \mathrm{d}\tau = \vec{F}_\tau . \tag{17}$$

We shall demonstrate that, in the reaction of annihilation the particle and his antiparticle, don't merely the energy and the momentum is preserved but also the mass. Consider a system composed by the particle and his antiparticle (with equal mass $m_1 = m_2 = m$) having the parameters $E_1, \vec{p}_1$ respective $E_2, \vec{p}_2$. According to the relation (12), the system mass is

$$m_{s.p} = \sqrt{2}\sqrt{m^2 + E_1 E_2 / c^4 - p_1 p_2 \cos\alpha / c^2} . \tag{18}$$

Through annihilation, the system of particles with rest mass is transformed in a system of photons. Considering a bi-photonic annihilation, the mass of the photons having the $E_{ph1}, \vec{p}_{ph1}$ respective $E_{ph2}, \vec{p}_{ph2}$, is, see relation (13),

$$m_{s.ph} = \sqrt{2E_{ph1}E_{ph2}\left(1-\cos\beta\right)} / c^2 = \sqrt{2p_{ph1}p_{ph2}\left(1-\cos\beta\right)} / c \neq 0 , \tag{19}$$

where β is the angle between the momentums of the two photons.

Using the conservation law of energy, for the annihilation reaction,

$$E_1 + E_2 = E_{ph1} + E_{ph2} \tag{20}$$

and the conservation law of momentum

$$\vec{p}_1^{\,2} + \vec{p}_2^{\,2} + 2p_1 p_2 \cos\alpha = \vec{p}_{ph1}^{\,2} + \vec{p}_{ph2}^{\,2} + 2p_{ph1}p_{ph2}\cos\beta , \tag{21}$$

through simple calculations is demonstrated that mass of two systems are equal

$$m_{s.p} = m_{s.ph} . \tag{22}$$

Based on this result we can demonstrate easy that is incorrect to assert that mass is changed to radiating energy [13]. A system of free particles characterized with parameters $m_i, E_i \,\square\, m_i c^2 + E_{ki}, \vec{p}_i$ (E_{ki} is the kinetic energy of the particle), in chaotic motion, and having the centre of mass in rest, has a mass

$$m_{s.p} = \sum_i \left(m_i + E_{ki}/c^2\right) > \sum_i m_i . \tag{23}$$

If the system of particles is cooled through caloric radiation (emission of photons) having the energy E_r, the kinetic energy of each particles diminish and therefore mass the system diminish with

$$\Delta m_{s.p} = \sum\left(\Delta E_{ki}/c^2\right) = E_r/c^2 . \tag{24}$$

According to the demonstration, the caloric radiation of a system of photons with the momentums orientated accidental, he has a non-vanishing mass. If we consider that the photons are emitted in couples with the equal momentums and orientate in opposite senses (because the centre of mass to remain in rest) then the angle $\beta = \pi$ and therefore the mass of a couples is $m_{s.ph.i} = 2E_{ph.i}/c^2$ and total mass the photons is $m_{s.ph.} = \sum\left(2E_{ph.i}/c^2\right) = E_r/c^2$. Results that the missed mass of the system of particles through cooling we can find out in mass of the system of emitted photons.

Conclusions

We can draw the conclusion that the energy and mass are different physical measures, between they existing just a quantitative relation. More, analyzing the definitions of the two measures, it can be observed that the properties are opposite. The inertial mass measure the property the system to oppose to the change of state, while the energy measures the property of the system produced the change of other system state. The option for mass in the proper system is compliant as much with the invariance requirements of ale quadri - dimensional formulation the relativity theory how much with: the conservation law of the mass for the systems of particles and the definitions of the lengths, durations and the kinematic measures derived from these.

References

1. Jammer, M. - *Concepts of mass*, Harvard Univ. Press, 1961
2. Okun, L.B. - *The Concept Of Mass*, Physics Today, June, 1989
3. Oas, G. - *On the Abuse and Use of Relativistic Mass*, arXiv: physics/0504110v2, 21 Oct. 2005
4. Okun, L.B. - The Concept of Mass in the Einstein Year, presented at *the 12th Lomonosov Conference on Elementary Particle Phyiscs*, Moscow State University, August 25-31, 2007, arXiv:hep-ph/0602037v1, 3 Feb. 2006
5. Simaciu, I., Moşescu, N. - Principiul lui Mach generalizat, *Buletinul de Fizică şi Chimie*, nr. XI, pp. 225-227, 1987
6. Higgs, P.W. - Broken Symmetries and the Masses of Gauge Bosons, *Phys. Rev. Lett*. 13, p. 508, 1964
7. Wilczek, F. - The Origin of Mass, *Modern Physics letters* A, Vol. 21, No. 9, pp. 701-712, 2006
8. Haisch, B., Rueda, A., Puthoff, E.H. - *Phys. Rev.* A., 49, 678, 1994
9. Simaciu, I. - Masa inerţială în fizică stocastică, Buletinul Universităţii Petrol-Gaze din Ploieşti, Seria Matematică-Informatică-Fizică, Vol. LIV, Nr. 1, 2002, pp. 61-64, 2002
10. Feynman, R., Leighton, R., Sands, M. - *The Feynman Lectures on Physics*, vol. I, Ch. 9, Addison Wesley, 1963
11. Landau, L., Lifchitz, E. - *Theorie des Champs,* Ed. Mir, Moscou, 1970
12. Simaciu, I., Ionescu-Pallas, N., Borsos, Z. - Fenomenele relativiste ca efecte ale modificării obiectelor, fenomenelor şi instrumentelor de măsură, Buletinul Universităţii Petrol-Gaze din Ploieşti, seria Matematică-Informatică-Fizică, Vol. LVII, nr. 3, 2005, pp. 69-74, 2005
13. Brehme, R.W. - The Advantage of Teaching Relativity with Four-Vector, *American Journal of Physics,* Vol. 36, No. 10, 1968

Problema relaţiei dintre masă şi energie

Rezumat

Lucrarea analizează concepţiile despre cele două mărimi fizice fundamentale: energia şi masa. În prima parte sunt prezentate concepţiile din fizica clasică. În partea a doua sunt prezentate concepţiile din fizica modernă: fizica relativistă şi fizica cuantică. În partea a treia sunt aduse noi argumente care susţin faptul

că mărimile sunt distincte calitativ şi relația lui Einstein este doar o relație cantitativă între masa în sistemul propriu şi energie. Cheia rezolvării problemei o constituie definiția relativistă a vitezei în conexiune cu definițiile relativiste ale spațiului parcurs şi a duratei în care a fost parcurs. Definițiile relativiste sunt compatibile numai în cazul vitezei proprii $\vec{\upsilon} = d\vec{r} / d\tau$ *. În acest caz impulsul relativist este produsul dintre masa în sistemul propriu şi viteza proprie* $\vec{p} = m\vec{\upsilon}_\tau = m\vec{\upsilon} \Big/ \sqrt{1-\upsilon^2/c^2}$.